

# Electroweak Physics for Color Superconductivity

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## Abstract

We construct the effective theories describing the electroweak interactions for the low energy excitations associated with the color superconductive phases of QCD at high matter density. The main result, for the 3 flavor case, is that the quasiparticle Goldstone boson  $\pi^0$  decay into two physical massless photons is identical to the zero density case once we use the new Goldstone decay constant and the modified electric charge  $\tilde{e} = e \cos \theta$ , with  $\tan \theta = 2e/\sqrt{3}g_s$  and  $g_s$  the strong coupling constant. For 2 flavors we find that the coupling of the quarks to the neutral vector boson  $Z^0$  is modified with respect to the zero density case. We finally point out possible applications of our result to the physics of compact objects.

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## I. INTRODUCTION

Recently quark matter at very high density has attracted a great flurry of interest [1–7]. In this limit, quark matter is expected to behave as a color superconductor [1,2]. Possible phenomenological applications are associated with the description of neutron star interiors, neutron star collisions and the physics near the core of collapsing stars.

In a superconductive phase, the color symmetry is spontaneously broken and a hierarchy of scales, for given chemical potential, is generated. Indicating with  $g_s$ , the underlying coupling constant, the relevant scales are: the chemical potential  $\mu$  itself, the dynamically generated gluon mass  $m_{gluon} \sim g_s \mu$  and the Gap parameter  $\Delta \sim \frac{\mu}{g_s} e^{-\frac{\alpha}{g_s}}$  with  $\alpha$  a calculable constant. Since for high  $\mu$  the coupling constant  $g_s$  (evaluated at the fixed scale  $\mu$ ) is  $\ll 1$ , we have:

$$\Delta \ll m_{gluon} \ll \mu . \quad (1.1)$$

Massless excitations dominate physical processes at very low energy with respect to the energy Gap ( $\Delta$ ). Their spectrum is intimately related to the underlying global symmetries and the way they are realized at low energies. They also obey low energy theorems governing their interactions which can be usefully encoded in effective low energy Lagrangians (like for cold and dilute QCD [8]). It is possible to order the effective Lagrangian terms describing the Golstone boson self interactions in number of derivatives. The resulting theory for dilute QCD is named Chiral Perturbation Theory [8]. Unfortunately this well defined scheme is not sufficient for a complete description of hadron dynamics since new massive hadronic resonances appear at relatively low energies like the  $\sigma$  or the vector  $\rho$  and new effective Lagrangians of the type described in [9] are needed.

Another set of relevant constraints is provided by 't Hooft anomaly matching conditions [10]. In Reference [11], it was shown that the low energy spectrum, at finite density, displays the correct quantum numbers to saturate the 't Hooft global anomalies. It was also observed that QCD at finite density can be envisioned, from a global symmetry and anomaly point of view, as a chiral gauge theory [12,13]. In Reference [14] it was demonstrated, using a variety of field theoretical tools, that 't Hooft anomaly matching conditions must hold for any cold but dense gauge theory.

In this paper we construct the electroweak interactions to complete the low energy effective theory describing Quantum Chromo Dynamics with two and three flavors at high density. This work can be considered as the first step for properly describing the phenomenology associated with electroweak processes stemming, for example, from the core

of some neutron stars which may be dense enough to be in a two, three or both flavor superconductive phases [15].

Some possible implications of electroweak interactions in compact objects for 3 flavors QCD at high matter density has been also partially investigated in [16].

The low energy theory for two flavors without considering electroweak interactions is provided in Ref. [17] while the one for three flavors has been developed in [18]. We use the non linear framework [19] to construct the low energy theories.

First for the three flavor case, we include the  $QED$  interactions and complete the low energy theory by implementing the global anomalies via the Wess-Zumino-Witten term. This topological term is essential when describing the time honored  $\pi^0 \rightarrow \gamma\gamma$  process at finite density [20].

By considering in detail the axial anomaly at the fundamental level and by comparing it with the gauged version of the Wess-Zumino term we demonstrate that the coefficient of the topological term is the same as zero density. The only place where the finite density effects enters are in the redefinition of the electric charge which is now given by  $e \cos \theta$  with  $\tan \theta = 2e/\sqrt{3}g_s$  and in the high density value of the Goldstone bosons decay constant. Our result stems directly from the Higgsing nature of the Color superconductive phase and it is at variance with the result presented in [20]. We briefly comment on the nature of the discrepancy.

Besides the importance of the  $\pi^0 \rightarrow \gamma\gamma$  process per se and its phenomenological consequences when investigating possible Quark-type stars featuring a CFL core, the topological coefficient term is also critical when considering the solitonic (Skyrme) solutions of the effective Lagrangian. In fact now we have the same winding number as for ordinary QCD and hence we get massive excitations, which after collective quantization, describe spin half particles with quantum numbers identical to the ordinary baryons. This is the actual realization of the Quark-Hadron continuity [6] advocated for the baryon-type sector of the Color-superconductive phase. We then introduce the electroweak interactions for 3 flavors.

Next we generalize the effective Lagrangian theory for 2 flavors presented in [17] to describe electroweak interactions. We observe that the Lagrangian not only reproduces the physical eigenstates for the photon and the eighth gluon [4] but also predicts a modified coupling of the quarks to the neutral vector boson  $Z^0$  with respect to the zero density case. The physical effect, which is our main result for the 2 flavor case, is not suppressed with respect to the zero density weak processes. On general grounds we expect this result to affect the Quark-stars (with a 2SC component) cooling processes [22] via neutrino emission. In particular we have in mind the neutrino transparency question directly related to the mean

free path of a neutrino in a compact star. This question has already attracted some interest [23]. Indeed for ordinary neutron stars [22] the mean free path is significantly affected by the *neutron* –  $\nu$  scattering reaction due to the presence of neutral currents. In a 2SC star we then expect the *quark* –  $\nu$  scattering to play an equally important role.

In Sect. II we study the anomalous process  $\pi^0 \rightarrow \gamma\gamma$  at high matter density, complete the effective Lagrangian for the quasiparticle Goldstone bosons by adding the Wess-Zumino term and comment on related issues. In Sect. III we gauge the weak sector for the three flavor case. In Sect. IV we briefly review the 2 flavor case while the generalization to include the electroweak processes is done in Sect. V. Finally in Sect. VI we summarize and conclude.

## II. QED FOR THE 3 FLAVOR CASE: THE $\pi_0 \rightarrow \gamma\gamma$ PROCESS

Let us start with the case of  $N_f = 3$  light flavors. At zero density only the confined Goldstone phase is allowed and the resulting symmetry group is  $SU_V(3) \times U_V(1)$ . Indeed there is no solution for the 't Hooft anomaly conditions with massless composite fermions leaving intact the flavor group. In this case the topological Wess-Zumino term for the Goldstone bosons is needed to implement the global anomalies of the underlying theory at the effective Lagrangian level. The Vafa-Witten theorem [24], valid for vector-like theories, prohibits the further breaking of the remaining vector-like symmetries like  $U_V(1)$ .

Turning on low baryon density we expect the theory to remain in the confined phase with the same number of Goldstone bosons (i.e. 8). Evidently the 't Hooft anomaly conditions are still satisfied. At very high density, dynamical computations suggest [3] that the preferred phase is a superconductive one and the following ansatz for a quark-quark type of condensate is energetically favored:

$$\epsilon^{\alpha\beta} \langle q_{L\alpha;a,i} q_{L\beta;b,j} \rangle \sim k_1 \delta_{ai} \delta_{bj} + k_2 \delta_{aj} \delta_{bi} , \quad (2.1)$$

and a similar expression holds for the right transforming fields. The Greek indices represent spin,  $a$  and  $b$  denote color while  $i$  and  $j$  indicate flavor. The condensate breaks the gauge group completely while locking the left/right transformations with color. The final global symmetry group is  $SU_{c+L+R}(3)$ , and the low energy spectrum consists of 9 Goldstone bosons. Before constructing the Lagrangian for the true massless Goldstones it is instructive to introduce the left and right transforming fields [18]:

$$L \rightarrow g_L L g_c , \quad R \rightarrow g_R R g_c , \quad (2.2)$$

where  $L/R$  parameterizes the Goldstone bosons induced by the appearance of the condensate (2.1).  $g_{L/R} \in SU_{L/R}(3)$  while  $g_c \in SU(3)$  of color. The covariant derivative describing color and electromagnetic interactions is

$$D_\mu L = \partial_\mu L - i e A_\mu Q L - i g_s G_\mu^m L T^m , \quad (2.3)$$

with  $Q = \text{diag}(2/3, -1/3, -1/3)$  the ordinary quark charges, and  $T^m$  the generators for color which, for convenience, are defined such that  $T^8 = \frac{\sqrt{3}}{2}Q$  and  $T^3 = \text{diag}(0, \frac{1}{2}, -\frac{1}{2})$ . Likewise for the  $R$  field. Here  $A$  denotes the standard photon field while  $G^m$  are the gluon fields. The vev in Eq. (2.1) (corresponding to  $L \propto \delta_{ic}$ ) locks together flavor and color and the Higgs mechanism sets in providing masses for all but one linear combination of the gluon  $G^8$  and photon  $A$ . The massive eigenstate can be easily identified by investigating the kinetic term

$$\text{Tr} [D_\mu L^\dagger D^\mu L] \quad (2.4)$$

which leads to a mass term of the type:

$$g_s^2 \frac{1}{2} \sum_{m=1}^7 G^{m2} + \text{Tr} [Q^2] \left( e A + \frac{\sqrt{3}}{2} g_s G^8 \right)^2 , \quad (2.5)$$

where for simplicity we have suppressed the Lorentz indices. We can now identify the massive  $\tilde{G}^8$  and the orthogonal massless eigenstate  $\tilde{A}$  via:

$$\tilde{G}^8 = \cos \theta G^8 + \sin \theta A , \quad (2.6)$$

$$\tilde{A} = -\sin \theta G^8 + \cos \theta A , \quad (2.7)$$

with

$$\cos \theta = \sqrt{3} \frac{g_s}{\sqrt{3g_s^2 + 4e^2}} , \quad \sin \theta = 2 \frac{e}{\sqrt{3g_s^2 + 4e^2}} . \quad (2.8)$$

$\tilde{A}$  is reinterpreted as the physical photon.

The same covariant derivative for the underlying quark fields can be compactly written as:

$$D_\mu = \partial_\mu - i e A_\mu Q \times \mathbf{1} - i g_s G_\mu^m \mathbf{1} \times T^m . \quad (2.9)$$

The first  $3 \times 3$  matrix is in flavor space while the second is in color. This notation is also convenient when investigating the global anomalies at the fundamental fermion level. The unbroken generator associated with the new photon  $\tilde{A}$  is, in flavor  $\times$  color space,

$$\tilde{Q} = Q \times \mathbf{1} - \mathbf{1} \times Q . \quad (2.10)$$

It is easy to check that all quarks, in medium, have integer charges in  $\tilde{e}$  units<sup>§</sup> and the condensate (2.1) is indeed neutral under  $\tilde{Q}$ .

The relevant terms in the covariant derivative can be rewritten as:

$$g_s G^8 \mathbf{1} \times T^8 + e A Q \times \mathbf{1} = g'_s \tilde{G}^8 \left[ \frac{\sqrt{2}}{2} \tilde{T}^8 - \frac{\sqrt{3}}{4} \tilde{Q} \cos(2\theta) \right] + \tilde{e} \tilde{A} \tilde{Q} , \quad (2.11)$$

where  $\tilde{e} = e \cos \theta$  and  $g'_s = g_s / \cos \theta$ . The second term corresponds to the unbroken  $U(1)$  gauge symmetry which we identify as the new QED with coupling  $\tilde{e}$ .  $\tilde{T}^8$  is the generator orthogonal to  $\tilde{Q}$ , normalized according to  $\text{Tr} [\tilde{T}^8 \tilde{T}^8] = 3/2$  and is given by:

$$\tilde{T}^8 = \frac{\sqrt{6}}{4} (Q \times \mathbf{1} + \frac{2\sqrt{3}}{3} \mathbf{1} \times T^8) \equiv \frac{\sqrt{6}}{4} (Q \times \mathbf{1} + \mathbf{1} \times Q) . \quad (2.12)$$

It is well known that anomalies play an important role for describing phenomenological processes like  $\pi_0 \rightarrow \gamma\gamma$  in cold and dilute matter. But before discussing the anomaly equation we should understand how the physical Goldstone bosons emerge. At high density, in the 3 flavor case, the symmetry group  $SU_L(3) \times SU_R(3) \times SU_c(3)$  breaks spontaneously to  $SU_{c+L+R}(3)$  leaving behind 16 Goldstone bosons. However, being  $SU_c(3)$  a gauge group 8 Goldstone bosons are absorbed in the longitudinal components of the massive gluons. So we are left with 8, not colored, physical massless Goldstone bosons. They can be encoded in the unitary matrix [18]

$$U = L R^\dagger , \quad (2.13)$$

transforming linearly under the left-right flavor rotations

$$U \rightarrow g_L U g_R^\dagger . \quad (2.14)$$

with  $g_{L/R} \in SU_{L/R}(N_f)$ . In our notation  $U$  is the transpose of  $\Sigma$  defined in Ref. [18].  $U$  satisfies the non linear realization constraint  $U U^\dagger = 1$ . We also require  $\det U = 1$ . In this way we avoid discussing the axial  $U_A(1)$  anomaly at the effective Lagrangian level. (See Ref. [25] for a general discussion of trace and  $U_A(1)$  anomaly). We have

$$U = e^{i \frac{\Phi}{F}} , \quad (2.15)$$

with  $\Phi = \sqrt{2} \Phi^a t^a$  representing the 8 Goldstone bosons.  $t^a$  are the standard generators of  $SU(3)$  (i.e.  $t^3 = \text{diag}(\frac{1}{2}, -\frac{1}{2}, 0)$ ), with  $a = 1, \dots, 8$  and  $\text{Tr} [t^a t^b] = \frac{1}{2} \delta^{ab}$ .  $F$  is the Goldstone bosons decay constant at finite density. Clearly  $\pi^0$  is associated with  $t^3$ .

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<sup>§</sup>Clearly with respect to the new photon field  $\tilde{A}$  the new charge units is  $e \cos \theta$  and the ratio between the quark and electron charge is integer.

Recently it has been shown in [11] that all of the interesting superconductive phases do respect, as for general chiral gauge theories at zero density, global anomaly matching conditions a la 't Hooft. In [14] it has been shown that anomaly matching conditions are unmodified with respect to the zero density case provided that one correctly implements the group theoretical structure of the global anomalies. At high density, in the 3 flavor case, using the fact [11,14] that the anomalous coefficient is unmodified by density effects, we have that the anomalous variation of the axial current  $j_5^{\mu 3}$  associated with  $\pi^0$  is given by:

$$\begin{aligned}\partial_\mu j_5^{\mu 3} &= -\frac{e^2}{16\pi^2} \epsilon^{\alpha\beta\mu\nu} F_{\alpha\beta} F_{\mu\nu} \text{Tr}[(t^3 \times \mathbf{1})(Q \times \mathbf{1})^2] \\ &= -\frac{e^2 N_c}{16\pi^2} \epsilon^{\alpha\beta\mu\nu} F_{\alpha\beta} F_{\mu\nu} \text{Tr}[t^3 Q^2] \\ &= -\frac{e^2}{32\pi^2} \epsilon^{\alpha\beta\mu\nu} F_{\alpha\beta} F_{\mu\nu} ,\end{aligned}\tag{2.16}$$

where  $N_c = 3$  comes from the trace over color space and  $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha = \cos\theta \tilde{F}_{\alpha\beta} + \sin\theta \tilde{G}_{\alpha\beta}^8$  with  $\tilde{F}_{\alpha\beta} = \partial_\alpha \tilde{A}_\beta - \partial_\beta \tilde{A}_\alpha$  and  $\tilde{G}_{\alpha\beta}^8 = \partial_\alpha \tilde{G}_\beta^8 - \partial_\beta \tilde{G}_\alpha^8$ . This leads to the following expression in terms of the physical vector eigenstates:

$$\begin{aligned}\partial_\mu j_5^{\mu 3} &= -\frac{e^2}{32\pi^2} \epsilon^{\alpha\beta\mu\nu} F_{\alpha\beta} F_{\mu\nu} \\ &= -\frac{e^2}{32\pi^2} \epsilon^{\alpha\beta\mu\nu} (\cos^2\theta \tilde{F}_{\alpha\beta} \tilde{F}_{\mu\nu} + 2\sin\theta \cos\theta \tilde{F}_{\alpha\beta} \tilde{G}_{\mu\nu}^8 + \sin^2\theta \tilde{G}_{\alpha\beta}^8 \tilde{G}_{\mu\nu}^8) .\end{aligned}\tag{2.17}$$

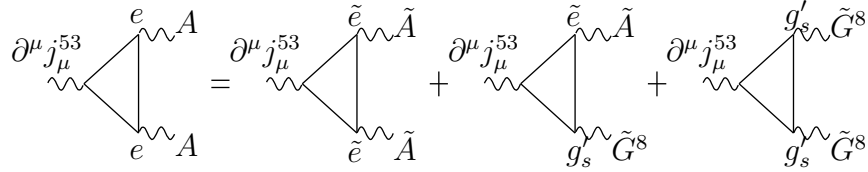


FIG. 1. Triangle anomaly for 3 flavors QCD at high density.

In figure 1 we graphically represent equation (2.17) via Feynman diagrams. We have  $g'_s = \frac{2\sqrt{3}}{3} \frac{e}{\sin\theta}$  and  $\tilde{e} = e \cos\theta$ . For each  $\tilde{A}$  we associate the generator  $\tilde{Q} = Q \times \mathbf{1} - \frac{2\sqrt{3}}{3} \mathbf{1} \times T^8$  and for  $\tilde{G}^8$  the generator  $\frac{\sqrt{2}}{2} \tilde{T}^8 - \frac{\sqrt{3}}{4} \tilde{Q} \cos(2\theta) = \cos^2\theta \mathbf{1} \times T^8 + \frac{\sqrt{3}}{2} \sin^2\theta Q \times \mathbf{1}$  as prescribed by the covariant derivative. Clearly the first diagram, in the right hand side, corresponds to  $\pi^0 \rightarrow \tilde{\gamma} \tilde{\gamma}$  in the superconducting phase where  $\tilde{\gamma}$  indicates the physical massless photon  $\tilde{A}$ . We conclude that the anomalous electromagnetic properties of the superconductive state are identical to the ones of ordinary QCD provided that we replace the electric charge  $e$  with  $\tilde{e} = e \cos\theta$ .

We are now ready to review the low energy effective theory for the 3 flavor case at high matter density. The effective Lagrangian at low energies [18] for the true massless Goldstone bosons is similar to the ordinary effective Lagrangian for QCD at zero density except for an extra Goldstone boson associated with the spontaneously broken  $U_V(1)$  symmetry which we will not consider for the moment.

The effective Lagrangian globally invariant under chiral rotations is (up to two derivatives and counting  $U$  as a dimensionless field)

$$L = \frac{F^2}{2} \text{Tr} [\partial_\mu U \partial^\mu U^\dagger] . \quad (2.18)$$

The Wess-Zumino term [26] can be compactly written by using the language of differential forms. It is useful to introduce the algebra valued Maurer-Cartan one forms:

$$\alpha = (\partial_\mu U) U^{-1} dx^\mu \equiv (dU) U^{-1} , \quad \beta = U^{-1} dU = U^{-1} \alpha U , \quad (2.19)$$

which transform, respectively, under the left and right  $SU(N_f)$  flavor group. The Wess-Zumino effective action is

$$\Gamma_{WZ} [U] = C \int_{M^5} \text{Tr} [\alpha^5] . \quad (2.20)$$

The price to pay in order to make the action local is to augment by one the space dimensions. Hence the integral must be performed over a five-dimensional manifold whose boundary ( $M^4$ ) is the ordinary Minkowski space. We now show that the constant  $C$  is the same as the one at zero density, i.e.:

$$C = -i \frac{N_c}{240\pi^2} , \quad (2.21)$$

where  $N_c$  is the number of colors (fixed to be 3 in this case). This is a consequence of Eq. (2.17). More specifically we now compare the current algebra prediction for the time honored process  $\pi^0 \rightarrow 2\tilde{\gamma}$  with the amplitude predicted using Eq. (2.20) by gauging the electromagnetic sector [27,28] of the Wess-Zumino term.

Before gauging the Wess-Zumino term we need to stress that in presence of an Higgsing phenomenon gauge symmetry is clearly not lost and one is *always* entitled to work with the un-rotated gauge fields and to rotate them to the mass eigenstates only in the very end.

In order to better understand the anomalies we recall that the fully gauged Wess Zumino term under the  $SU_L(N_f) \times SU_R(N_f)$  chiral symmetry group is:

$$\Gamma_{WZ} [U, A_L, A_R] = \Gamma_{WZ} [U] + 5Ci \int_{M^4} \text{Tr} [A_L \alpha^3 + A_R \beta^3]$$



$$\begin{aligned}
& -5C \int_{M^4} \text{Tr} [(dA_L A_L + A_L dA_L)\alpha + (dA_R A_R + A_R dA_R)\beta] \\
& +5C \int_{M^4} \text{Tr} [dA_L dU A_R U^{-1} - dA_R dU^{-1} A_L U] \\
& +5C \int_{M^4} \text{Tr} [A_R U^{-1} A_L U \beta^2 - A_L U A_R U^{-1} \alpha^2] \\
& +\frac{5C}{2} \int_{M^4} \text{Tr} [(A_L \alpha)^2 - (A_R \beta)^2] + 5Ci \int_{M^4} \text{Tr} [A_L^3 \alpha + A_R^3 \beta] \\
& +5Ci \int_{M^4} \text{Tr} [(dA_R A_R + A_R dA_R)U^{-1} A_L U - (dA_L A_L + A_L dA_L)U A_R U^{-1}] \\
& +5Ci \int_{M^4} \text{Tr} [A_L U A_R U^{-1} A_L \alpha + A_R U^{-1} A_L U A_R \beta] \\
& +5C \int_{M^4} \text{Tr} [A_R^3 U^{-1} A_L U - A_L^3 U A_R U^{-1} + \frac{1}{2}(U A_R U^{-1} A_L)^2] \\
& -5Cr \int_{M^4} \text{Tr} [F_L U F_R U^{-1}] , \tag{2.22}
\end{aligned}$$

with the two-forms  $F_L$  and  $F_R$  defined as  $F_L = dA_L - iA_L^2$  and  $F_R = dA_R - iA_R^2$  and the one form  $A_{L/R} = A_{L/R}^\mu dx_\mu$ .  $r$  is a real arbitrary parameter. The previous Lagrangian, when identifying the vector fields with true gauge vectors, correctly saturates the underlying global anomalies [28].

The last term in Eq. (2.22) is a gauge covariant term. However it is not invariant under parity\*\* and so the parameter  $r$  must vanish. All the other terms are related by gauge invariance.

We can now restrict the attention to the electromagnetic interactions by constraining the vector fields to satisfy the equation

$$A_L = A_R = Q A = e Q A_\mu dx^\mu , \tag{2.23}$$

with  $A_\mu$  the ordinary photon field. The Wess-Zumino term collapses to the following [28] form:

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\*\* For reader's convenience we provide the rules for parity transformation

$$\begin{aligned}
A_{L,R}(\vec{x}) &\leftrightarrow A_{R,L}(-\vec{x}) , & U(\vec{x}) &\leftrightarrow U^{-1}(-\vec{x}) , \\
\alpha &\leftrightarrow -\beta , & d &\leftrightarrow d , & (\text{measure}) &\leftrightarrow -(\text{measure}) ,
\end{aligned}$$

as well as for charge conjugation

$$A_{L,R} \leftrightarrow -A_{R,L}^T , \quad U \leftrightarrow U^T , \quad \alpha \leftrightarrow \beta^T .$$

$$\Gamma_{WZ}[U, A] = \Gamma_{WZ}[U] + 5eCi \int_{M^4} A \text{Tr} [Q(\alpha^3 + \beta^3)] - 10e^2C \int_{M^4} AdA \text{Tr} \left[ Q^2(\alpha + \beta) + \frac{1}{2}(QU^{-1}QdU - QUQdU^{-1}) \right] . \quad (2.24)$$

The last term leads to the following  $\pi_0 \rightarrow \gamma\gamma$  Lagrangian:

$$\mathcal{L}_{\pi_0 \rightarrow \gamma\gamma} = -\frac{30}{4}e^2C i \text{Tr} [t^3 Q^2] \pi^0 \frac{\sqrt{2}}{F} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} , \quad (2.25)$$

and  $\pi^0 = \Phi^3$ . The last ingredient is the axial current (expanded up to the first derivative) expressed as function of the Goldstone bosons:

$$j_5^{\mu a} = -\frac{F}{\sqrt{2}} \partial^\mu \Phi^a . \quad (2.26)$$

One pole saturation of the 3 point function anomalous amplitude [29], when compared with the underlying anomalous variation in Eq. (2.17) leads straightforwardly to the result for  $C$  in Eq. (2.21).

One can also fix the Wess-Zumino coefficient by matching at the effective Lagrangian level the anomalous variation for the left or right currents associated with the non abelian *consistent*<sup>††</sup> anomalies.

This procedure seems to be the one adopted in Ref. [20]. However in Ref. [20] the anomaly coefficient (which should be the same for all of the currents, left and right) differs from our canonical one by a factor 3. The reason is that the physical pions (i.e. quasiparticle) are colorless and hence a factor 3 (due to the color factor) should appear in the anomalous coefficient.

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<sup>††</sup>We differentiate here consistent anomaly from the covariant anomaly [30]. The two anomalies are related. In order to compare directly the axial anomalous variation without using pole saturation it is convenient to consider the anomalies in Bardeen's form. This is achieved by subtracting to the Wess Zumino displayed in (2.22) the action  $\Gamma_c = \Gamma_{WZ}[U = 1; A_L, A_R]$  [28]. The new action term (which clearly does not change the  $\pi^0 \rightarrow \gamma\gamma$  process) reads

$$\Gamma'_{WZ}[U; A_L, A_R] = \Gamma_{WZ}[U; A_L, A_R] - \Gamma_{WZ}[1; A_L, A_R] .$$

If we restrict to the electromagnetic variations one gets [28]

$$\delta(\Gamma'_{WZ}) = \frac{30}{2}e^2C i \int_{M^4} \epsilon_5^3 \text{Tr} [t^3 Q^2] \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} = - \int_{M^4} \epsilon_5^3 \partial_\mu j_5^{\mu 3} ,$$

which leads again to the result for  $C$  obtained in the main text.

More specifically, it seems to us, that in [20] the anomalous equation for the left transforming currents (which can be easily connected to ours) involving directly the physical photon field is not properly implemented. Indeed, as we have shown in Eq. (2.17) at the fundamental quark level, we can simply relate the axial anomaly between the physical photon  $\tilde{A}$  and  $A$ . This is due to the fact that we have an Higgsing of the gauge symmetries.

The consequences of our result are two-fold. The first is that the rate of the  $\pi^0 \rightarrow \tilde{\gamma}\tilde{\gamma}$  is augmented by an order of magnitude ( $N_c^2$ ) with respect to reference [20]. This fact can be relevant for the physics of Quark-type stars featuring a CFL core.

The second is related to the solitonic (Skyrme) solutions of the effective Lagrangian. In fact now we have the same winding number as for ordinary QCD and hence we get massive excitations, which after collective quantization, describe spin half particles with the same quantum numbers of ordinary baryons. This is the actual realization of the Quark-Hadron continuity [6] advocated for the baryon-type sector of the Color-superconductive phase.

To the previous Lagrangian one can still add the extra Goldstone boson associated to the  $U_V(1)$  symmetry breaking without altering the previous discussion (see [18]). One can check that the global anomalies are correctly implemented by carefully gauging the Wess-Zumino term [27,28] with respect to the flavor symmetries. Hence for the 3 flavor case too, the 't Hooft global anomalies are matched at finite (low and high) density.

In writing the Goldstone Lagrangian we have not yet considered the breaking of Lorentz invariance at finite density. Following Ref. [18] we note that the Goldstones obey, in medium, a linear dispersion relation of the type  $E = v|\vec{p}|$ , where  $E$  and  $|\vec{p}|$  are respectively the energy and the momentum of the Goldstone bosons. We can simply include the Lorentz breaking by generalizing the effective Lagrangian (2.18) in the following way:

$$L = \frac{F^2}{2} \text{Tr} \left[ \dot{U}\dot{U}^\dagger - v^2 \vec{\nabla} U \cdot \vec{\nabla} U^\dagger \right] . \quad (2.27)$$

Clearly by rescaling the vector coordinates  $\vec{x} \rightarrow \vec{x}/v$  we can recast the previous Lagrangian in the Eq. (2.18) form.

An important feature is that  $\alpha$ , being a differential form, is unaffected by coordinate rescaling (actually topological terms being independent on the metric are unaffected by medium effects), and hence the Wess-Zumino term is not modified at finite matter density.

Due to the breaking of the baryon number the final global symmetry group in the superconductive phase differs from the ordinary Goldstone phase.

### III. ELECTROWEAK FOR 3 FLAVORS AT HIGH MATTER DENSITY

Next we extend the previous effective Lagrangian by incorporating the electroweak intermediate vector mesons as external fields. We adopt a standard procedure which has been often employed in literature. An example is the effective Lagrangian theories used to describe a strong electroweak sector (technicolor like theories) [31]. A more closely related example is the description of radiative and weak processes for low energy QCD at zero temperature and matter density in the framework of Chiral Perturbation Theory [32].

According to this procedure we need to partially gauge the flavor subgroup by generalizing the effective Lagrangian in the following way<sup>‡‡</sup>:

$$\begin{aligned}
 DU &= \partial U - i \frac{g}{\sqrt{2}} [W^+ \tau^+ + W^- \tau^-] U - i \frac{g}{\cos \theta_W} Z^0 [\tau^3 U - \sin^2 \theta_W [Q, U]] \\
 &\quad - ie A [Q, U] \\
 &= \partial U - i \frac{g}{\sqrt{2}} [W^+ \tau^+ + W^- \tau^-] U - i \frac{g}{\cos \theta_W} Z^0 [\tau^3 U - \sin^2 \theta_W [Q, U]] \\
 &\quad - i \tilde{e} \tilde{A} [Q, U] - i \tilde{e} \tan \theta \tilde{G}^8 [Q, U]
 \end{aligned} \tag{3.1}$$

where  $\theta_W$  is the electroweak angle and

$$\tau^+ = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \tau^- = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \tau^3 = \frac{1}{2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \tag{3.2}$$

The last two terms in Eq. (3.1) describe the, non anomalous, interaction of the Goldstone bosons with respectively the physical massless photon and the physical massive gluon.

In the present framework the  $SU_L(2)$  subgroup of the  $SU_L(3)$  flavor group is completed gauged. At the quark level this allows us to describe the weak interactions for the up and down quarks in medium. Indeed it is easy to recognize that the upper bidimensional subblocks of the matrices given in Eq. (3.2) are the standard  $SU_L(2)$  generators. For the strange quark we considered just the diagonal interactions, conveniently encoded in the last line of

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<sup>‡‡</sup> A word of caution is needed when describing non leptonic weak decays. In this case we expect, as for the zero density case, that the strong interactions may affect the electroweak processes. Now one can first integrate out the heavy electroweak intermediate vector meson fields by constructing a new (weak) effective Lagrangian [32]. The price to pay in this way is the proliferation of unknown coefficients.

the matrix  $\tau^3$ . This is because the charm quark is not included in the low energy Lagrangian we are considering.

Clearly  $\tilde{e} = e \cos \theta$  is the finite density new electric charge.

As for dilute QCD, when  $U$  is evaluated on the vev we have a contribution to the masses of the  $W$  and  $Z$  that we do not consider [32]. We also expect a non zero mixing among the gluons and the weak gauge bosons, which will be explored in more detail for the 2 flavor case and that will be shown to be small. One can study this mixing, in more detail, for the 3 flavor case by generalizing the covariant derivative in Eq. (2.3) for the  $L/R$  fields.

For the massive quark case, we have to include the Cabibbo-Kobayashi-Maskawa mixing angles. However it is straightforward to generalize the previous expression to contain the mixing angles<sup>§§</sup>.

We finally observe that the electroweak physics for 3 flavors at high matter density is similar to the zero density QCD except for a new photon electric charge [32] and the explicit presence of a massive gluon. The latter couples to the Goldstone bosons via the standard charge operator  $Q$ .

#### IV. REVIEW OF THE 2 FLAVOR LOW ENERGY EFFECTIVE THEORY

QCD with 2 flavors has gauge symmetry  $SU_c(3)$  and global symmetry

$$SU_L(2) \times SU_R(2) \times U_V(1) . \quad (4.1)$$

At very low baryon density it is reasonable to expect that the confined Goldstone phase persists. However at very high density, it is seen, via dynamical calculations [1,2], that the ordinary Goldstone phase is no longer favored compared with a superconductive one associated with the following type of diquark condensates:

$$\langle L^{\dagger a} \rangle \sim \langle \epsilon^{abc} \epsilon^{ij} q_{Lb,i}^\alpha q_{Lc,j;\alpha} \rangle , \quad \langle R^{\dagger a} \rangle \sim -\langle \epsilon^{abc} \epsilon^{ij} q_{Rb,i;\dot{\alpha}} q_{Rc,j}^{\dot{\alpha}} \rangle , \quad (4.2)$$

---

<sup>§§</sup> For example  $\tau^+$  now becomes

$$\tau^+ = \begin{pmatrix} 0 & V_{ud} & V_{us} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (3.3)$$

with  $V$  mixing angles (see [32]).

$q_{Lc,i;\alpha}$ ,  $q_{Rc,i}^{\dot{\alpha}}$  are respectively the two component left and right spinors.  $\alpha, \dot{\alpha} = 1, 2$  are spin indices,  $c = 1, 2, 3$  stands for color while  $i = 1, 2$  represents the flavor. If parity is not broken spontaneously, we have

$$\langle L_a \rangle = \langle R_a \rangle = f \delta_a^3, \quad (4.3)$$

where we choose the condensate to be in the 3rd direction of color. The order parameters are singlets under the  $SU_L(2) \times SU_R(2)$  flavor transformations while possessing baryon charge  $\frac{2}{3}$ . The vev leaves invariant the following symmetry group:

$$[SU_c(2)] \times SU_L(2) \times SU_R(2) \times \tilde{U}_V(1), \quad (4.4)$$

where  $[SU_c(2)]$  is the unbroken part of the gauge group. The  $\tilde{U}_V(1)$  generator  $\tilde{B}$  is the following linear combination of the previous  $U_V(1)$  generator  $B = \frac{1}{3} \text{diag}(1, 1, 1)$  and the broken diagonal generator of the  $SU_c(3)$  gauge group  $T^8 = \frac{1}{2\sqrt{3}} \text{diag}(1, 1, -2)$ :\*\*\*

$$\tilde{B} = B - \frac{2\sqrt{3}}{3} T^8. \quad (4.5)$$

The quarks with color 1 and 2 are neutral under  $\tilde{B}$  and consequently the condensate too<sup>††</sup>.

The superconductive phase for  $N_f = 2$  possesses the same global symmetry group of the confined Wigner-Weyl phase [11].

The dynamics of the Godstone bosons can be efficiently encoded in a non linear realization framework as presented in [17]. In this framework the relevant coset space is  $G/H$  with  $G = SU_c(3) \times U_V(1)$  and  $H = SU_c(2) \times \tilde{U}_V(1)$  is parameterized by [17]:

$$\mathcal{V} = \exp(i \xi^i X^i), \quad (4.6)$$

where  $\{X^i\}$   $i = 1, \dots, 5$  belong to the coset space  $G/H$  and are taken to be  $X^i = T^{i+3}$  for  $i = 1, \dots, 4$  while

$$X^5 = B + \frac{\sqrt{3}}{3} T^8 = \text{diag}\left(\frac{1}{2}, \frac{1}{2}, 0\right). \quad (4.7)$$

$T^a$  are the standard generators of  $SU(3)$ . The coordinates

$$\xi^i = \frac{\Pi^i}{f} \quad i = 1, 2, 3, 4, \quad \xi^5 = \frac{\Pi^5}{\tilde{f}}, \quad (4.8)$$

---

\*\*\*Nota Bene: for 2 flavors we keep the standard definition for the color generators.

<sup>†††</sup> $\tilde{B}$  is  $\sqrt{2}\tilde{S}$  of Ref. [17]

via  $\Pi$  describe the Goldstone bosons. The vevs  $f$  and  $\tilde{f}$  are expected, when considering asymptotically high densities [7], to be proportional to  $\mu$ .

$\mathcal{V}$  transforms non linearly:

$$\mathcal{V}(\xi) \rightarrow u_V g \mathcal{V}(\xi) h^\dagger(\xi, g, u) h_V^\dagger(\xi, g, u) , \quad (4.9)$$

with

$$u_V \in U_V(1) , \quad g \in SU_c(3) , \quad h(\xi, g, u) \in SU_c(2) , \quad h_{\tilde{V}}(\xi, g, u) \in \tilde{U}_V(1) . \quad (4.10)$$

The linear realizations are related via [17] :

$$V_a = \frac{L_a + R_a}{\sqrt{2}} = \sqrt{2} f e^{i \frac{\Pi^5}{f}} \mathcal{V}^{-1^3}_a . \quad (4.11)$$

The  $V_a$  field explicitly describes the vev properties and, as expected, transforms under the underlying gauge transformations as a diquark.

It is convenient to define the Hermitian (algebra valued) Maurer-Cartan one-form

$$\omega_\mu = i \mathcal{V}^\dagger D_\mu \mathcal{V} \quad \text{with} \quad D_\mu \mathcal{V} = (\partial_\mu - i g_s G_\mu) \mathcal{V} , \quad (4.12)$$

with gluon fields  $G_\mu = G_\mu^m T^m$  while  $g_s$  is the strong coupling constant.  $\omega$  transforms as:

$$\omega_\mu \rightarrow h(\xi, g, u) \omega_\mu h^\dagger(\xi, g, u) + i h(\xi, g, u) \partial_\mu h^\dagger(\xi, g, u) + i h_{\tilde{V}}(\xi, g, u) \partial_\mu h_{\tilde{V}}^\dagger(\xi, g, u). \quad (4.13)$$

Following [17] we decompose  $\omega_\mu$  into

$$\omega_\mu^\parallel = 2 S^a \text{Tr} [S^a \omega_\mu] \quad \text{and} \quad \omega_\mu^\perp = 2 X^i \text{Tr} [X^i \omega_\mu] , \quad (4.14)$$

where  $S^a$  are the unbroken generators of  $H$  with  $S^{1,2,3} = T^{1,2,3}$ ,  $S^4 = \tilde{B}/\sqrt{2}$ . Summation over repeated indices is assumed.

The most generic two derivative kinetic Lagrangian is [17].

$$L = f^2 a_1 \text{Tr} [\omega_\mu^\perp \omega^{\mu\perp}] + f^2 a_2 \text{Tr} [\omega_\mu^\perp] \text{Tr} [\omega^{\mu\perp}] , \quad (4.15)$$

The presence of a double trace term is due to the absence of the traceless condition for the broken generator  $X^5$  and it emerges naturally in the non linear realization framework at the same order (in derivative expansion) with respect to the single trace term. This is not the case for the linearly realized effective Lagrangian [33,34].

### A. In Medium Fermions

Following Ref. [17] we define:

$$\tilde{\psi} = \mathcal{V}^\dagger \psi, \quad (4.16)$$

transforming as  $\tilde{\psi} \rightarrow h_{\tilde{V}}(\xi, g, u)h(\xi, g, u)\tilde{\psi}$  and  $\psi$  possesses an ordinary quark transformations (as Dirac spinor). This construction mimics the Heavy Quark Effective formalism [35].

The whole [17] non linearly realized effective Lagrangian describing in medium fermions, gluons and their self interactions, up to two derivatives is:

$$\begin{aligned} \mathcal{L} = & f^2 a_1 \text{Tr} [\omega_\mu^\perp \omega^{\mu\perp}] + f^2 a_2 \text{Tr} [\omega_\mu^\perp] \text{Tr} [\omega^{\mu\perp}] \\ & + b_1 \bar{\tilde{\psi}} i \gamma^\mu (\partial_\mu - i \omega_\mu^\parallel) \tilde{\psi} + b_2 \bar{\tilde{\psi}} \gamma^\mu \omega_\mu^\perp \tilde{\psi} \\ & + m_M \bar{\tilde{\psi}} \gamma^5 (iT^2) \tilde{\psi} \varepsilon^{ij} + \text{h.c.}, \end{aligned} \quad (4.17)$$

where  $\tilde{\psi}^C = i \gamma^2 \tilde{\psi}^*$ ,  $i, j = 1, 2$  are flavor indices and

$$T^2 = S^2 = \frac{1}{2} \begin{pmatrix} \sigma^2 & 0 \\ 0 & 0 \end{pmatrix}, \quad (4.18)$$

$a_1$ ,  $a_2$ ,  $b_1$  and  $b_2$  are real coefficients while  $m_M$  is complex. For later convenience, it is convenient, to express the third and forth terms as

$$b_1 \bar{\tilde{\psi}} i \gamma^\mu (\partial_\mu - i \omega_\mu) \tilde{\psi} + (b_2 - b_1) \bar{\tilde{\psi}} \gamma^\mu \omega_\mu^\perp \tilde{\psi} = b_1 \bar{\tilde{\psi}} i \gamma^\mu D_\mu \tilde{\psi} + (b_2 - b_1) \bar{\tilde{\psi}} \gamma^\mu \omega_\mu^\perp \tilde{\psi}. \quad (4.19)$$

From the last two terms, representing a Majorana mass term for the quarks, we deduce that the massless degrees of freedom are the  $\psi_{a=3,i}$  which possess the correct quantum numbers to match the 't Hooft anomaly conditions [11]. To the previous general effective Lagrangian we should also add the  $SU(2)$  gluon kinetic term.

Following Ref. [18] the breaking of Lorentz invariance to the  $O(3)$  subgroup can easily be taken into account by providing different coefficients to the temporal and spatial indices of the Lagrangian, i.e.:

$$\begin{aligned} \mathcal{L} = & f^2 a_1 \text{Tr} [\omega_0^\perp \omega_0^\perp - \alpha_1 \vec{\omega}^\perp \vec{\omega}^\perp] + f^2 a_2 [\text{Tr} [\omega_0^\perp] \text{Tr} [\omega_0^\perp] - \alpha_2 \text{Tr} [\vec{\omega}^\perp] \text{Tr} [\vec{\omega}^\perp]] \\ & + b_1 \bar{\tilde{\psi}} i [\gamma^0 (\partial_0 - i \omega_0^\parallel) + \beta_1 \vec{\gamma} \cdot (\vec{\nabla} - i \vec{\omega}^\parallel)] \tilde{\psi} + b_2 \bar{\tilde{\psi}} [\gamma^0 \omega_0^\perp + \beta_2 \vec{\gamma} \cdot \vec{\omega}^\perp] \tilde{\psi} \\ & + m_M \bar{\tilde{\psi}} \gamma^5 (iT^2) \tilde{\psi} + \text{h.c.}, \end{aligned} \quad (4.20)$$

where the new coefficients  $\alpha$ s and  $\beta$ s encode the effective breaking of Lorentz invariance and the flavor indices are omitted.



## V. ELECTROWEAK INTERACTIONS IN MATTER

To construct the low energy effective theory for the electroweak sector we need to gauge the weak isospin as well as the hypercharge sector and finally identify the correct massless as well massive eigenstates.

We identify the  $SU_L(2)$  generators  $\tau_L^i = \sigma_L^i/2$  with  $i = 1, 2, 3$  and  $\sigma^i$  the standard Pauli's matrices as the weak generators. The hypercharge is, in full generality,  $Y = \tau_R^3 + \frac{B-L}{2}$  with  $L$  the lepton number while  $\tau_R$  labels the  $SU_R(2)$  generators. Quarks and leptons have standard charges, in particular  $B = 1/3$  for quarks.

We need to generalize the one form  $\omega_\mu = i\mathcal{V}^\dagger D_\mu \mathcal{V}$  by introducing the new covariant derivative:

$$D_\mu \mathcal{V} = (\partial_\mu - ig_s G_\mu - ig' Y B_\mu^y) \mathcal{V} = (\partial_\mu - ig_s G_\mu - ig' \frac{B}{2} B_\mu^y) \mathcal{V} . \quad (5.1)$$

$B_\mu^y$  is the standard hypercharge gauge field. Neglecting, for the moment, the breaking of Lorentz invariance and substituting Eq. (5.1) in Eq. (4.15) we have the following quadratic terms:

$$a_1 g_s^2 \frac{v^2}{2} \sum_{i=4}^7 G_\mu^i G^{i\mu} + (a_1 + 2a_2) \frac{f^2}{2} \left[ \frac{g_s}{\sqrt{3}} G_\mu^8 + \frac{g'}{3} B_\mu^y \right]^2 . \quad (5.2)$$

We now rewrite the previous terms using the electroweak eigenstates associated with the photon field  $A_\mu$  and the neutral massive vector boson  $Z_\mu^0$ .

$$B_\mu^y = \cos \theta_W A_\mu - \sin \theta_W Z_\mu^0 , \quad (5.3)$$

with  $\theta_W$  the standard electroweak angle. Focusing only on the second term in Eq. (5.2) one has:

$$(a_1 + 2a_2) \frac{f^2}{2} \left\{ \left[ \frac{g_s}{\sqrt{3}} G^8 + \frac{e}{3} A \right]^2 - \frac{2}{3} e \tan \theta_W Z^0 \left[ \frac{g_s}{\sqrt{3}} G^8 + \frac{e}{3} A \right] + \tan^2 \theta_W \frac{e^2}{9} Z^{02} \right\} , \quad (5.4)$$

where  $e = g' \cos \theta_W$  is the standard electric charge and for simplicity we have dropped the Lorentz indices. To this term we have to add the electroweak quadratic mass term for  $Z^0$

$$\frac{1}{2} m_Z^2 Z^{02} . \quad (5.5)$$

The new massless eigenstate is interpreted as the, in medium, photon and is given by:

$$\tilde{A}_\mu = \cos \theta_Q A_\mu - \sin \theta_Q G_\mu^8 , \quad (5.6)$$

with

$$\cos \theta_Q = \sqrt{3} \frac{g_s}{\sqrt{3g_s^2 + e^2}} , \quad \sin \theta_Q = \frac{e}{\sqrt{3g_s^2 + e^2}} . \quad (5.7)$$

The massive state orthogonal to  $\tilde{A}_\mu$  is:

$$\tilde{G}_\mu^8 = \cos \theta_Q G_\mu^8 + \sin \theta_Q A_\mu . \quad (5.8)$$

Using the new base, the complete tree level quadratic term involving  $\tilde{G}^8$  and  $Z^0$  is

$$\begin{aligned} (a_1 + 2a_2) \frac{f^2}{18} \left\{ (3g_s^2 + e^2) \tilde{G}^{82} - 2e \tan \theta_W Z^0 \tilde{G}^8 \sqrt{3g_s^2 + e^2} + \tan^2 \theta_W e^2 Z^{02} \right\} \\ + \frac{m_Z^2}{2} Z^{02} . \end{aligned} \quad (5.9)$$

By diagonalizing the previous matrix we have the new massive eigenstates:

$$\begin{aligned} \hat{G}_\mu^8 &= \cos \theta_M \tilde{G}_\mu^8 + \sin \theta_M Z_\mu^0 , \\ \tilde{Z}_\mu^0 &= \cos \theta_M Z_\mu^0 - \sin \theta_M \tilde{G}_\mu^8 , \end{aligned} \quad (5.10)$$

with

$$\tan 2\theta_M = \frac{2f^2 \tan \theta_W (a_1 + 2a_2) \sqrt{3g_s^2 + e^2}}{(a_1 + 2a_2) f^2 [\tan^2 \theta_W e^2 - (3g_s^2 + e^2)] + 9m_Z^2} \approx \frac{2f^2 \tan \theta_W}{9m_Z^2} (a_1 + 2a_2) \sqrt{3g_s^2 + e^2} , \quad (5.11)$$

where in the last step we have considered the physical limit  $m_Z^2 \gg f^2$ . In the same limit, as expected,  $\tilde{G}^8$  and  $Z^0$  do not mix much and we can use them as physical eigenstates.

Having identified the correct physical eigenvalues and eigenvectors we are now ready to consider the Lagrangian for the quarks. The relevant Lagrangian terms are:

$$b_1 \bar{\psi} i \gamma^\mu D_\mu \psi + (b_2 - b_1) \bar{\psi} \gamma^\mu \omega_\mu^\perp \tilde{\psi} , \quad (5.12)$$

where we generalize the covariant derivative to describe the weak interactions in the following way:

$$\begin{aligned} D_\mu &= \partial_\mu - i \frac{g}{\sqrt{2}} [W_\mu^+ \tau_L^+ \times \mathbf{1} + W_\mu^- \tau_L^- \times \mathbf{1}] - i \frac{g}{\cos \theta_W} Z_\mu^0 [\tau_L^3 \times \mathbf{1} - \sin^2 \theta_W Q \times \mathbf{1}] \\ &\quad - ie A_\mu Q \times \mathbf{1} - ig_s \sum_{m=1}^8 G_\mu^m \mathbf{1} \times T^m . \end{aligned} \quad (5.13)$$

We adopted a notation similar to the one used in the 3 flavor case; i.e.  $\text{flavor}_{2 \times 2} \times \text{color}_{3 \times 3}$ . We also have

$$Q = \tau^3 + \frac{B - L}{2} , \quad (5.14)$$

with  $\tau = \tau_L + \tau_R$ . Considering the gluon-photon mixing we have:

$$D_\mu = \dots - i\tilde{e}\tilde{A}_\mu\tilde{Q} - i\frac{g_s}{\cos\theta_Q}\tilde{G}_\mu^8 [\cos\theta_Q^2 \mathbf{1} \times T^8 + \sqrt{3}\sin\theta_Q^2 Q \times \mathbf{1}] , \quad (5.15)$$

where the dots represent the terms unchanged in Eq. (5.13).  $\tilde{e}$  is the new electric charge

$$\tilde{e} = e \cos\theta_Q , \quad (5.16)$$

$\tilde{Q}$  is the new electric charge operator associated with the field  $\tilde{A}_\mu$ :

$$\tilde{Q} = \tau^3 \times \mathbf{1} + \frac{\tilde{B} - L}{2} = Q \times \mathbf{1} - \frac{1}{\sqrt{3}} \mathbf{1} \times T^8 , \quad (5.17)$$

The quarks that acquires a mass term (i.e. the ones in the color direction one and two) have half integer charges under  $\tilde{Q}$  while the massless quarks (the ones in direction three of color) have the ordinary proton and neutron charges in units of  $\tilde{e}$ .

To the leading order the term containing  $\omega^\perp$ , in Eq. (5.12), does not involve any electric interaction but it does affect the neutral weak currents. Expanding around  $\mathcal{V} = 1$  one gets:

$$(b_2 - b_1)\bar{\psi}\gamma^\mu \left( g_s \sum_{m=4}^7 G_\mu^m T^m + \frac{g_s}{\sqrt{3}\cos\theta_Q}\tilde{G}_\mu^8 X^5 - \frac{g}{3}\frac{\sin\theta_W^2}{\cos\theta_W}Z_\mu^0 X^5 \right) \psi + \dots , \quad (5.18)$$

where the dots stand for terms involving higher terms in the expansion of  $\mathcal{V}$ . The neutral weak current is directly affected by finite density effects even when neglecting the small physical mixing between the eighth gluon  $\tilde{G}^8$  and  $Z^0$ . We also observe that the modified electroweak coupling only emerges for quarks with color indices 1 and 2, since  $X^5 = \frac{1}{2}\text{diag}(1, 1, 0)$ . The new term does not affect the light quarks with color index 3. In particular this effect depends crucially upon the unknown ratio:  $(b_2 - b_1)/b_1$ , and depends on  $\mu$ . The full modified quark coupling to the neutral weak current is now:

$$b_1 \frac{g}{\cos\theta_W} Z_\mu^0 \bar{\psi}\gamma^\mu \left[ T_L^3 - \sin\theta_W^2 Q - \frac{b_2 - b_1}{3b_1} \sin\theta_W^2 (B + Q - \tilde{Q}) \right] \psi , \quad (5.19)$$

where we used Eq. (5.17) and  $X^5 = B + Q - \tilde{Q}$ .

The modification of the neutral current coupling to the quarks at high matter density constitutes our main result for what concerns the 2 flavors case.

We can point immediately to relevant phenomenological consequences of our result. The cooling history of compact objects, like for instance the ones generated as remnant (proto-star) of a Supernova explosion [22], is heavily related to the neutrino physics. The neutrino transparency of a given star [22] can significantly modify the cooling history and is associated with the mean free path of a neutrino in the star. Indeed for ordinary neutron stars [22] the mean free path is strongly affected by the *neutron* –  $\nu$  scattering reaction due to the presence of neutral currents. In a 2SC star we then expect the *quark* –  $\nu$  scattering to play an equally important role and its modification with respect to the zero density case to be dictated by the new weak coupling of Eq. (5.19).

Finally, on general grounds, we have shown that a direct phenomenological evidence for color superconductivity can be, in principle, found by studying the neutral weak interactions associated with a given high matter density physical system.

## VI. CONCLUSIONS

We investigated the electroweak interactions for the relevant color superconductivity phases. More specifically we included the *QED* interactions at the effective Lagrangian level for the three flavor case at high density developed in [18]. We identified the physical photon and provided the Wess-Zumino term needed for describing the  $\pi^0 \rightarrow \gamma\gamma$  interaction in medium.

We found, as main result for the 3 flavor case, that at high matter density the quasiparticle Goldstone boson  $\pi^0$  decay into two physical massless photons is identical to the zero density case once we replace the electric charge with the modified charge  $\tilde{e} = e \cos \theta$  and also use the new decay constant. This result is consistent with the underlying global anomaly constraints at finite density [11,14]. However it differs from the conclusions drawn in [20]. We finally extended the effective Lagrangian to include the weak interactions.

For the 2 flavor case we constructed the effective Lagrangian theory containing the proper electroweak theory. We, first, identified the physical photon. We have also discovered that our Lagrangian not only correctly yields the physical eigenstates for the photon and the eight gluon but also predicts that  $Z^0$  has a modified coupling to the quarks (in the direction 1 and 2 of color) with respect to the zero density case. The physical effect is not suppressed with respect to zero density weak processes. The effect, which is our main result for the 2 flavor case, is not suppressed with respect to the zero density weak processes. On general grounds we expect this result to affect the Quark-stars (with a 2SC component) cooling processes [22] via neutrino emission. The neutrino transparency question is indeed directly

related to the mean free path of a neutrino in a compact star. In neutron stars [22] the mean free path is known to be significantly affected by the *neutron* –  $\nu$  scattering reaction due to neutral currents. Likewise in a 2SC star the *quark* –  $\nu$  scattering will play a relevant role with the new weak coupling displayed in Eq. (5.19). Clearly a dynamical computation of the quantity  $(b_2 - b_1)/b_1$ , although beyond the goal of this paper, might be very interesting.

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